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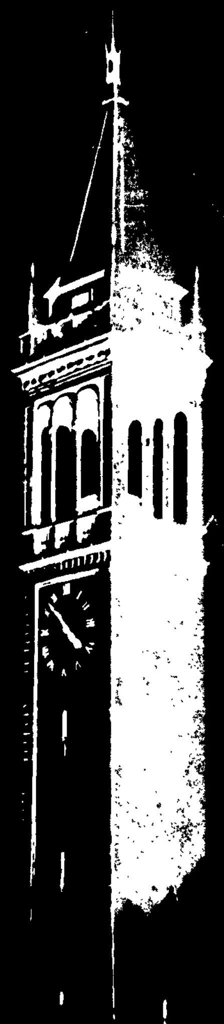

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A Model for Quality Control

by

L. Breiman

Series No. 60, Issue No. 438
Contract No. Nonr-222(53)
October 1, 1962

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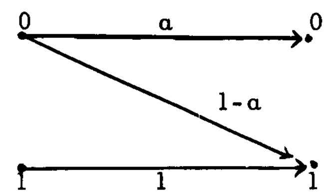
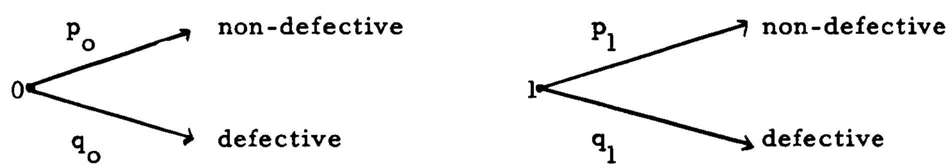
1. <u>Introduction</u>. Our purpose herein is to introduce a model for quality control, and to characterize a policy which maximizes the given payoff function. Envisage a machine with two internal states, 0 and 1. Starting at state 0 at time zero, it manufactures an item which is either defective or non-defective and then in unit time, either remains in state 0 or goes to state 1 according to the scheme.



Note that 1 is an absorbing state. After every transition the machine manufactures another item. There are different probabilities of the item being defective or non-defective according as the state of the machine, and given by:



where $p_o > p_1$, $\alpha < 1$.

After any number of items have been turned out, the machine may be stopped for an integer time T, and when this repair period T is over it is in state 0 and the manufacturing process begins again.

If there is profit C for every non-defective item, cost D for every defective item and a charge G for every time unit that the machine is in repair, then roughly, we wish to maximize the long run profit per unit time.

By a policy is meant some method of deciding when to stop the machine and repair it. The difficulty, of course, is that we cannot directly observe the state of the machine, but only the condition of the manufactured items. If our stop and repair policy is too erratic, it may be that the limiting cost per unit time does not exist. We extricate ourselves from this difficulty by considering only "sensible" policies Our main result is, that among these latter, the optimum is of the form: there is a number λ^* such that when the conditional probability that the machine is currently in state 1, given all the observed items up to the present, exceeds λ^*, stop and repair. This policy does not seem to reduce to any of the standard quality control techniques.

This problem is similar to those treated by Howard [1], except that the pertinent Markov chain has an infinite number of states. We were led to it by our work on stopping rules, and the treatment is an interesting example of some of the techniques mentioned in [2]. In the section to follow we reduce the problem to a stopping rule problem. Following that procedure we show that the stopping rule problem has solutions of the desired form. This result is summarized in Theorem 2 of section 4.

2. Notations and Reduction of Problem. The class S of policies we will work within are defined by: a policy is in S if

i) it leads to stop and repair infinitely often with probability one.

ii) the decision as to whether to stop and repair is based only on the sequence of items turned out since the end of the previous repair period.

iii) if a sequence of items, following some repair period, leads to a decision to stop and repair, then this same sequence following any repair period leads to that decision.

(iv) the expected duration of running time between repair periods is finite.

There is possible here an alternative approach which starts from a much larger class of policies and shows that for each policy in the larger class there is a policy in S that is at least as good. See Blackwell [3], for instance. But this would take us into theory far afield from the present example.

Under a policy in S, define the first cycle of the machine to be its history from time zero to the end of the first repair period, but not including the first item manufactured after repair. The second cycle begins with this latter item and extends similarly to the end of the second repair period, and so on. Let R_k be the total profit during the k^{th} period, N_k the length of the k^{th} period. Then the profit per unit time over the first n cycles is

$$\frac{R_1+\ldots+R_n}{N_1+\ldots+N_n} .$$

Under a policy in S, $R_1, \ldots, R_n$ are independent, identically distributed, with $E|R_1| < \infty$, and similarly for the $N_1, \ldots, N_n$. The law of large number is thus in force so that with probability one

$$\lim_n \frac{R_1+\ldots+R_n}{N_1+\ldots+N_n} = \frac{ER_1}{EN_1} .$$

This relationship reduces the problem to an analysis of only the first cycle, i. e., to find a policy which maximizes ER_1/EN_1. For the real valued parameter β, define

$$\phi(\beta) = \sup(ER_1 - \beta EN_1)$$

where the sup is over all policies in S. Note that $\phi(\beta)$ is decreasing in β, since $ER_1-\beta EN_1$ is decreasing in β for every fixed policy. Also $\phi(\beta) > -\infty$, all β, since $ER_1 - \beta EN_1 > -\infty$ for the policy: stop and repair after one item. Let $\beta_o = \inf\{\beta;\phi(\beta) < \infty\}$, then

Proposition 1: $\beta_o < \infty$. There is a unique number $\beta^* > \beta_o$ such that $\phi(\beta^*) = 0$ and $\beta^* \geq C-(C+D)q_1$.

Proof: Suppose $\beta > C$, then the maximum amount we can make in any period is C, but because we are being charged an amount β for every period (because of the term $-\beta EN_1$) it follows that $\phi(\beta) < 0$, $\beta > C$. For each policy in S, $ER_1 - \beta EN_1$ is linear in β, hence $\phi(\beta)$ is concave on (β_o, ∞) and thus continuous. Since $\phi(\beta_o) = \infty$, for any given number M there is a policy such that $ER_1 - \beta_o EN_1 \geq 2M$. By continuity, there is an $\mathcal{E} > 0$ such that $ER_1 - (\beta_o + \mathcal{E})EN_1 \geq M$, so that $\phi(\beta_o + \mathcal{E}) > 0$. Since $\phi(\beta)$ is decreasing and concave there must be a unique solution β^* of $\phi(\beta) = 0$.

Now assume that $\beta^* < C-(C+D)q_1$ and considera policy that continues for n items, n large, and then stops and repairs. Since the machine is in state 1 with probability tending to unity as more and more transitions go by, ER_1 is equal to $(Cp_1 - Dq_1)n$ plus terms of lower order in n. But $Cp_1 - Dq_1 = C-(C+D)q_1$ so that $ER_1 - \beta^* EN_1$ is equal to $[C-(C+D)q_1 - \beta]n$ plus terms of lower order, contradicting $\phi(\beta^*) = 0$.

For this number β^*, we have $\sup(ER_1 - \beta^* EN_1) = 0$, so that $ER_1 - \beta^* EN_1 \leq 0$ for all policies in S. Hence

$$\sup \frac{ER_1}{EN_1} = \beta^* .$$

Further, if there is a policy which achieves the optimization of $ER_1 - \beta^* EN_1$, then this same policy optimizes ER_1/EN_1.

Let $\hat{R}$ be the profit from the items manufactured under a given policy and N the number of items manufactured.

$$R_1 = \hat{R} - GT$$

$$N_1 = \hat{N} + (T-1)$$

and $ER_1 - \beta^* EN_1 = E\hat{R} - \beta^* E\hat{N} - [GT + \beta^*(T-1)]$. Define random variables X_k by

$$X_k = \begin{cases} C \text{ if } k^{th} \text{ item is non-defective} \\ \\ -D \text{ if } k^{th} \text{ item is defective} \end{cases}$$

and using J to denote $\hat{R}-\beta^*\hat{N}$, we write

$$EJ = E \sum_{k-1}^{\hat{N}} (X_k - \beta^*)$$

By an interchange of summation and integration, valid since $E\hat{N} < \infty$ (see Doob [5], for instance), this becomes

$$EJ = \sum_{k=1}^{\infty} \int_{\{\hat{N} \geq k\}} (X_k - \beta^*) dP$$

Define $U_k = E(X_k | X_{k-1}, \ldots, X_1)$, then since the sets $\{\hat{N} \geq k\}$ depend only on $X_{k-1}, \ldots, X_1$. We rewrite again

$$EJ = \sum_{k=1}^{\infty} \int_{\{\hat{N} \geq k\}} \{U_k - \beta^*\} \, dP$$

if we put $V_k = P(X_k = -D | X_{k-1}, \ldots)$, then

$$U_k = C(1-V_k) - DV_k,$$

so

$$EJ = E \sum_{k=1}^{\hat{N}} (-(C+D)V_k + C - \beta^*)$$

The situation of maximizing EJ may be described as follows: for the k^{th} item we receive a fee

$$f(V_k) = -(C+D)V_k + C - \beta^*$$

and are free to stop at this point or to go one more item.

Parts of the above reduction, using cycles, to a stopping rule problem have been used before in other contexts, and the appropriate references are in [2].

3. Reduction to a Functional Equation

The next pertinent fact is:

Proposition 3: The V_k form a stationary Markov chain on $[q_o, q_1]$ such that if

$$F_1(v) = \frac{v(q_1 + \alpha q_o) - \alpha q_1 q_o}{v}$$

$$F_2(v) = \frac{v(\alpha p_o - q_1) + q_1(1 - \alpha p_o)}{1-v}$$

then if $V_k = v$, V_{k+1} is either $F_1(v)$ or $F_2(v)$ with probabilities v, 1-v.

Proof: We have that

$$V_{k+1} = P(X_{k+1} = -D \mid X_k, \ldots, X_1)$$

$$= \begin{cases} P(X_{k+1} = -D \mid X_k = -D,\ X_{k-1}, \ldots, X_1), & \text{probability } V_k \\ P(X_{k+1} = -D \mid X_k = C,\ X_{k-1}, \ldots, X_1), & \text{probability } 1-V_k \end{cases}$$

We introduce variables Y_k, U_k defined by: Y_k is the state of the machine just prior to the manufacture of the k^{th} item, and $U_k = P(Y_k \mid X_{k-1}, \ldots, X_1)$. Denoting $\gamma_1 = P(X_{k+1} = -D \mid X_k = -D,\ X_{k-1}, \ldots)$, $\gamma_2 = P(X_{k+1} = -D \mid X_k = C, \ldots, X_1)$, there follows

$$\gamma_1 = P(X_{k+1} = -D,\ X_k = -D \mid X_{k-1}, \ldots)/V_k$$

$$\gamma_2 = P(X_{k+1} = -D,\ X_k = C \mid X_{k-1}, \ldots)/1-V_k\ .$$

Thus,

$$\gamma_1 V_k = P(X_{k+1} = -D,\ X_k = -D \mid Y_k = 1)U_k + P(X_{k+1} = -D,\ X_k = -D \mid Y_k = 0)(1-U_k)$$

$$= q_1^2 U_k + q_o[\alpha q_o + (1-\alpha)q_1](1-U_k)$$

$$= (q_1 + \alpha q_o)(q_1 - q_o)U_k + \alpha q_o^2 + (1-\alpha)q_1 q_o$$

$$\gamma_2(1-V_k) = P(X_{k+1} = -D,\ X_k = C \mid Y_k = 1)U_k + P(X_{k+1} = -D,\ X_k = C \mid Y_k = 0)(1-U_k)$$

$$= p_1 q_1 U_k + p_o[\alpha q_o + (1-\alpha)q_1]\ (1-U_k)$$

$$= (\alpha p_o - q_1)(q_1 - q_o)U_k + \alpha p_o q_o + (1-\alpha)p_o q_1$$

We have, to boot, the relation

$$V_k = q_o(1-U_k) + q_1 U_k,$$

and solving this for U_k and substituting above yields the given expression for F_1, F_2. For $v \in [q_o, q_1]$, consider the functional equation for H

$$\text{(A)} \qquad H(v) = \max\left\{0, E[H(V_2) + f(V_2) \mid V_1 = v]\right\}$$

where $f(v) = -(C+D)v + C - \beta^*$. This equation may be derived in a fashion similar to that used by Bellman in dynamic programming problems. Heuristically, let H (v) be the maximum payoff starting from $V_1 = v$. We have our choice of stopping and receiving zero or of making the transition to V_2 where we receive the amount $f(V_2)$ plus our maximum expected payoff starting from V_2, this latter being $H(V_2)$.

However, the above heuristic does not establish any optimality properties, and while the connection between functional equations and optimal policies has been investigated, (see [4], for example) none of the results seem appropriate for the present problem. Therefore, we

must delve into the theory with the following theorem.

Theorem 1: If equation (A) has a bounded solution on $[q_o, q_1]$, let s^* denote the policy: stop when $V_k \epsilon \{v; H(v) = 0\}$. If $s^* \epsilon S$, then s^* is optimal in S (where here S denotes all policies with finite expected stopping time).

Proof: Let s be any policy in S with stopping variable $\hat{N}$ such that the set $\{\hat{N}=k\}$ depends only on the values of $X_1, \ldots, X_k$ and $E\hat{N} < \infty$. For any $\mathcal{E} > 0$, we may take n so large that

$$|EJ - \sum_1^n \int_{\{\hat{N} \geq k\}} f(V_k)dP - \int_{\{\hat{N} \geq n\}} H(V_k)dP| \leq \mathcal{E} .$$

Thus,

$$EJ \leq \mathcal{E} + \sum_1^{n-1} \int_{\{\hat{N} \geq k\}} f(V_k)dP + \int_{\{\hat{N} \geq n\}} [H(V_k)+f(V_k)]dP.$$

The set $\{\hat{N} \geq n\}$ depends only on $X_1, \ldots, X_{n-1}$ so we may replace the last integrand above by $E[H(V_n) + f(V_n) | V_{n-1}]$ and by (A) then

$$EJ \leq \mathcal{E} + \sum_1^{n-1} \int_{\{\hat{N} \geq k\}} f(V_k)dP + \int_{\{\hat{N} \geq n\}} H(V_{n-1})dP$$

But $\{\hat{N} \geq n\} \subset \{\hat{N} \geq n-1\}$ and $H(v) \geq 0$, so

$$EJ \leq \mathcal{E} + \sum_1^{n-1} \int_{\{\hat{N} \geq k\}} f(V_k)dP + \int_{\{\hat{N} \geq n-1\}} H(V_{n-1})dP.$$

Continuing to proceed this way, and noting that $V_1 = q_o$, we get

$$EJ \leq \mathcal{E} + H(q_o) + f(q_o) .$$

On the other hand, let N^* be the stopping variable given by s^*, and J^*, the payoff. Then,

$$EJ^* \geq -\mathcal{E} + \sum_{1}^{n-1} \int_{\{N^* \geq k\}} f(V_k)dP + \int_{\{N^* \geq n\}} E[H(V_k) + f(V_n) | V_{n-1}]\, dP$$

by (A), on $\{N^* > n-1\}$, the last integrand is equal to $H(V_{n-1})$, yielding

$$EJ^* \geq -\mathcal{E} + \sum_{1}^{n-1} \int_{\{N^* \geq k\}} f(V_k)dP + \int_{\{N^* \geq n\}} H(V_{n-1})dP.$$

Furthermore, on the set $\{N^* = n-1\}$, $H(V_{n-1}) = 0$, giving

$$EJ^* \geq -\mathcal{E} + \sum_{1}^{n-1} \int_{\{N^* \geq k\}} f(V_k)dP + \int_{\{N^* \geq n-1\}} H(V_{n-1})dP.$$

Continuing, we conclude $EJ^* \geq -\mathcal{E} + H(q_o) + f(q_o)$, which proves the theorem.

4. Solution of the Function Equation

To investigate the solution of (A), we first prove:

Proposition 2: If $\theta(v)$ is monotonic nonincreasing on q_o, q_1, then so is

$$E(\theta(V_2) | V_1 = v).$$

Proof: Let $\theta_{v_o}(v)$ be defined as

$$\theta_{v_o}(v) = \begin{cases} 1 & q_o \leq v \leq v_o \\ 0 & v_o < v \leq q_1 \end{cases}$$

Then,

$$E(\theta_{v_o}(V_2) | V_1 = v) = \theta_{v_o}(F_1(v))v + \theta_{v_o}(F_2(v))(1-v).$$

It is easy to verify that $F_1(v)$, $F_2(v)$ are monotonically increasing in v and that on $[q_o, q_1]$, $F_1(v) \geq F_2(v)$. Therefore,

$$E(\theta_{v_o}(V_2)|V_1=v) = \begin{cases} 1, & F_1(v) \le v_o \\ 1-v, & F_1(v) > v_o,\ F_2(v) \le v_o \\ 0, & F_2(v) > v_o \end{cases}$$

and is decreasing. Since every nonincreasing function can be arbitrarily closely approximated by finite sums with positive coefficients of functions of the type θ_{v_o}, the proposition follows.

To try and solve (A) we use an approximation procedure, defined by

$$H^{(n+1)}(v) = \max\left\{0, E[H^{(n)}(V_2)+f(V_2)|V_1=v]\right\}$$

with $H^{(1)}(v) \equiv 0$.

<u>Proposition 3. The $H^{(n)}(v)$ are a nondecreasing sequence of continuous functions on $[q_o, q_1]$.</u>

<u>Proof</u>: Assume that $H^{(n)}(v) \ge H^{(n-1)}(v)$. Then,

$$\begin{aligned} H^{(n+1)}(v) &= \max\left\{0, E[H^{(n)}(V_2)+f(V_2)|V_1=v]\right\} \\ &\ge \max\left\{0, E[H^{(n-1)}(V_2)+f(V_2)|V_1=v]\right\} \\ &= H^{(n)}(v) \ . \end{aligned}$$

And since $H^{(2)}(v) \ge 0$, we have always $H^{(n+1)}(v) \ge H^{(n)}(v)$. Furthermore, if $H^{(n)}(v)$ is continuous, then since $E(\theta(V_2)|V_1=v)$ is continuous if θ is continuous, the proposition holds.

<u>Proposition 4. $H(v) = \lim_n H_n(v)$ is a bounded solution of (A).</u>

<u>Proof</u>: Consider the function $av+b$, where

$$a = -\frac{\alpha}{1-\alpha}(C+D)$$

and b is taken so that $av+b \ge 0$, all $v \in [q_o, q_1]$. By a quick computation

$$E(V_2|V_1=v) = \alpha v + (1-\alpha)q_1$$

so

$$E[aV_2 + b + f(V_2) \mid V_1 = v] = (a - C - D)E(V_2 \mid V_1 = v) + b + C - \beta^*$$

$$= (a - C - D)[\alpha v + (1-\alpha)q_1] + b + C - \beta^*$$

$$= av + b + C - (C+D)q_1 - \beta^*$$

$$\leq av + b$$

This last by Proposition 1. Therefore, if $H^{(n)}(v) \leq av+b$, then

$$H^{(n+1)}(v) = \max\{0, E[H^{(n)}(V_2) + f(V_2) \mid V_1 = v]\}$$

$$\leq \max\{0, av+b\} = av+b.$$

This establishes that H(v) is bounded. That it is a solution is quite evident.

At this point, we have all the material necessary for our main result.

<u>Theorem 2</u>: Either the policy: never stop and repair, yields a larger ER_1/EN_1 then any policy in S, or there is a number $\lambda^* < 1$ such that the policy: stop and repair when

$$P(Y_k = 1 \mid X_k, \ldots, X_1) \geq \lambda^*$$

is in S and is optimal in S.

<u>Proof</u>: Let $S_n \subset [q_o, q_1]$ be defined by

$$S_n = \{v; H^{(n)}(v) = 0\} \quad .$$

By Propositions 2 and 3, S_n is of the form $[v_n, q_1]$, $v_n \leq q_1$, or empty. Since $F_1(q_1) = F_2(q_1) = q_1$, (A) gives $H(q_1) = \max[0, H(q_1) + g(q_1)]$ and since $g(q_1) = C-(C+D)q_1 - \beta^* \leq 0$, S_n is non-empty unless possibly $\beta^*_\infty = C-(C+D)q_1$. Leaving this latter case for the nonce, $S = \{v;\ H(v) = 0\} = \bigcap_1^\infty S_n$ is thus a set of the form $[\gamma, q_1]$. By Theorem 1, if the policy: stop when $V_k \in [\gamma, q_1]$, is in S, then it is optimum in S. Assume first that $\gamma < q_1$, and let N^* be the stopping variable. Then

$$P(N^* > n) \leq P(V_n \in' [\gamma, q_1]) \, .$$

To continue this inequality, we use

$$EV_n \leq q_1 P(V_n \epsilon [\gamma, q_1]) + \gamma P(V_n \epsilon [q_o, \gamma)),$$

To get

$$P(V_n \epsilon '[\gamma, q_1]) \leq \frac{q_1 - EV_n}{q_1 - \gamma} .$$

By their definitions,

$$EV_n = q_1 P(Y_n = 1) + q_o P(Y_n = 0)$$

and since $P(Y_n=0) = a^n$, this gives

$$EV_n = q_1 - (q_1 - q_o) a^n.$$

Substituting,

$$P(N^* > n) \leq \frac{q_1 - q_o}{q_1 - \gamma} \quad a^n$$

so $EN^* < \infty$. Now we show that if $\beta^* > C-(C+D)q_1$, then $\gamma < q_1$. For taking, limits in (A) as v goes up to q_1 yields

$$H(q_1-) = \max [\ 0, H(q_1-) + g(q_1)]$$

and $g(q_1) < 0$ implies $H(q_1-) = 0$. Therefore, we can certainly find a neighbor-hood of q_1, say $[q_1-\mathcal{E}, q_1]$, $\mathcal{E} > 0$, on which

$$H(F_1(v))v + H(F_2(v))(1-v) + g(v) \leq 0,$$

and in this neighborhood, then, $H(v) = 0$.

Now for the case $\beta^* = C-(C+D)q_1$. In this case,

$$\sup ER_1/EN_1 = C-(C+D)q_1.$$

But this is exactly the payoff from the policy that never stops.

The theorem is stated in terms of the variables $P(Y_k=1 | X_k, X_{k-1}, \ldots)$. These are related to the V_k variables by

$$V_{k+1} = q_1 P(Y_k=1 | X_k, \ldots) + [q_1(1-\alpha) + q_o \alpha][1 - P(Y_k=1 | X_k, \ldots)]$$

$$= \alpha(q_1 - q_o) P(Y_k=1 | X_k, \ldots) + q_1(1-\alpha) + q_o \alpha.$$

This transformation takes q_1 into 1 and $\gamma < q_1$ into some number $\lambda^* < 1$, concluding the proof of the theorem.

5. The Character of the Optimal Policy

We first give a more explicit form of the optimal policy by evaluating $P(Y_k=1 | X_k, X_{k-1}, \ldots)$. Note that

$$P(Y_k, Y_{k-1}, \ldots, Y_1, X_k, \ldots, X_1) = P(X_k, \ldots, X_1 | Y_k, \ldots, Y_1) P(Y_k, \ldots, Y_1)$$

$$= \prod_{j=1}^{k} P(X_j | Y_j) P(Y_k, \ldots, Y_1).$$

Define Q_j by

$$Q_j = P(Y_k=1, \ldots, Y_{j+1}, Y_j=0, \ldots, Y_1=0), \quad j=1, \ldots, k$$

so

$$Q_j = \begin{cases} (1-\alpha)\alpha^{j-1}, & j < k \\ \alpha^{k-1}, & j = k. \end{cases}$$

Then

$$P(Y_k=1, X_k, \ldots, X_1) = \sum_{j=1}^{k-1} \left[\prod_{r=1}^{j} P(X_r | Y_r=0) \prod_{r=j+1}^{k} P(X_r | Y_r=1) \right] Q_j$$

Let N_j = no. of defectives in the first j trials, so

$$P(Y_k=1, X_k, \ldots, X_1) = \sum_{j=1}^{k+1} q_o^{N_j} p_o^{j-N_j} q_1^{N_k - N_j} p_1^{k-j-(N_k-N_j)} Q_j.$$

Denoting $z = q_o p_1 / p_o q_1$, $w = p_o Q / p_1$.

$$P(Y_k=1, X_k, \ldots, X_1) = p_1^k \left(\frac{q_1}{p_1}\right)^{N_k} \frac{1-\alpha}{\alpha} \sum_{j=1}^{k-1} z^{N_j} w^j$$

Similarly

$$P(X_k, \ldots, X_1) = p_1^k \left(\frac{q_1}{p_1}\right)^{N_k} \frac{1-\alpha}{\alpha} \left[\sum_{j=1}^{k-1} z^{N_j} w^j + \frac{1}{1-\alpha} z^{N_k} w^k \right].$$

The optimal policy becomes: stop when

$$\sum_{j=1}^{k-1} z^{N_j} w^j \geq \lambda^{**} z^{N_k} w^k$$

where $\lambda^{**} = \lambda^* / (1-\alpha)(1-\lambda^*)$. Or, if J_j is the no. of defectives in the <u>last</u> j trials, stop when

$$\sum_{j=1}^{k-1} \left(\frac{1}{z}\right)^{J_j} \left(\frac{1}{w}\right)^j \geq \lambda^{**} \quad .$$

Or, if I_j is the no. of non-defectives in the last j trials, stop when

$$\sum_{j=1}^{k-1} \left(\frac{q_1}{q_o}\right)^{J_j} \left(\frac{p_1}{p_o}\right)^{I_j} \alpha^{-j} \geq \lambda^{**} .$$

While this above expression may or may not be interesting, a more illuminating form of the optimal policy was suggested by Roy Radner. This is: stop when

$$\frac{P(Y_k=1 \mid X_k, \ldots, X_1)}{P(Y_k=0 \mid X_k, \ldots, X_1)} \geq \lambda^{***}$$

The expression on the left is a likelihood ratio, and the policy may be stated as: at every step, test the hypothesis that the machine is in state one vs state 0, given all the relevant information. When the hypothesis can be accepted at a certain level, stop and repair.

The parameter λ^* seems difficult to compute, although some approximation methods are useful. As to the shortcomings of the model, they are more or less apparent, and it is our hope that more realistic models will follow.

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